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TECHNICAL NOTE

Bubble detachment criteria

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1. INTRODUCTION

In a paper [1] entitled "Das Abreissen von Dampfblasen an festen Heizflächen" (detachment of vapour bubbles from solid heating surfaces) published in this journal in 1983, I considered the detachment of a vapour bubble growing on a horizontal heater wall. Recently, van der Geld [2] criticized the results and raised some doubts concerning the detachment criterion given in ref. [1]. After a comparison of the force balance for a growing vapour bubble, well known from the literature and derived again in his paper, with the force balance in my paper he concluded that the corrected buoyancy term is missing in my equations and the detachment criterion is wrong. He stated (page 655, beginning of right column): "M's criterion thus leads to

$$\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=0$$

which can never be satisfied."

In this citation, M stands for Mitrovic, R_1 and R_2 are the mean radii of curvature of the interface at the bubble foot, while σ denotes the surface tension. The left-hand-side of the relationship thus represents the Laplace pressure.

The statement cited is the essence of van der Geld's paper [2], the other remarks, being less serious than they appear, will not be mentioned here. However, his explanations (page 654, right column, beginning of third paragraph) dealing with the position of the system boundary, as, "The thing that is observed to detach from a plate is the fluid boundary or, to put it differently, Gibb's dividing surface," should not pass this paper without a comment. We frequently use the concept of nonmaterial dividing surface introduced by Gibbs (not Gibb, as it stands in ref. [2], meant is Josiah Willard Gibbs, 1839–1903) when dealing with capillary systems. But the position of such a surface can be chosen almost arbitrarily within the interphase region. In any case, however, the thing that is observed to detach is, of course, a material body, the vapour that forms the bubble. The confusion in ref. [2] is perhaps due to vapour transparency. To make the vapour visible, we may replace it by liquid and reverse the action of gravity. Such a body represents a pendent drop which detaches in the same manner as the bubble, but cannot be identified with Gibbs' dividing surface.

The criticism in ref. [2] is based on an unrealistic hypothesis of the detachment process and on misinterpretation of my results. Although explained to some extent in ref. [1], a frozen bubble shape during detachment is not possible and the detachment process manifests itself even by a change of the shape, the author [2] did not take any notice of this matter and assumes again the bubble shape to be frozen. So far his criticism is irrelevant and unfounded. His statement (introduction in ref. [2]), "It was attempted in the article [1] . . . , to set up the governing force balance and to devise a detachment criterion on the basis of a force balance, irrespective of the orientation of the heated plane wall with respect to gravity," contradicts the facts. Actually, the considerations in ref. [1] are restricted to a bubble adhering to a horizontal wall. This is indicated in some figures by the vector $\boldsymbol{g}$ of gravitational acceleration which is perpendicular to the wall, see for example Fig. 4 in ref. [1]. Obviously, the author [2] realized this restriction, because, while summarizing the results of [1], he writes below equation (4): "Note that only bubbles are considered that are symmetrical around a vertical axis."

The original article [1] is written in German and it seems therefore appropriate to summarize the results first.

According to the literature in 1983, when the article [1] was published, the detachment diameter of a vapour bubble was determined from a force balance for an adhering bubble. Such a balance included both static and dynamic terms; the bubble detachment conditions, however, were not formulated. The shape of the detaching bubble was considered as unchanged, that is, 'frozen', and the bubble thus detaches itself like a rigid body. Furthermore, the surface tension was acting in the force balance as an adhering force, but at the same time also as a lifting force in most papers. The vapour bubble was then usually approximated by a truncated sphere, so that the surface tension did not actually have any influence on bubble detachment size.

The results of the paper [1] can briefly be stated as follows:

(1) It was shown that the assumption of the 'frozen' shape of a vapour bubble during the detachment process contradicts physical reality.

(2) The criteria that must be fulfilled if a bubble of 'frozen' shape nevertheless detaches itself from a horizontal wall were discussed. As shown in the paper, the detachment of a bubble of frozen shape requires that the excess force acting on the area formed between the vapour and the wall disappears.

(3) Finally, it was illustrated in the paper that only a change of the bubble shape leads to bubble detachment.

The considerations in the present paper are restricted to the above mentioned disappearance of the Laplace pressure at the moment of bubble detachment. It will be demonstrated that in the case of a frozen bubble shape, the excess force arising from the Laplace pressure which acts on the contact area between vapour and wall, has to vanish at the moment of bubble detachment. The other remarks in ref. [2], so far of a broader interest, will be included in a separate paper which is in preparation.

2. DETACHMENT CONDITION OF A BODY

2.1. *Detachment of a rigid body*

A vapour bubble, whose shape does not change during the detachment process, detaches from a wall like a rigid body,

NOMENCLATURE

F_a	force
G	weight force
K_0	reaction force
ΔK	excess force
p	pressure
r	radius.

Greek symbol

σ	surface tension.

Subscripts

G	gas (vapour)
L	liquid
0	at wall.

and the detachment process obeys the principles of classical mechanics. For illustration, we first consider a rigid body lying on the surface of a horizontal wall, Fig. 1. All forces exerted on the body are parallel to the weight G and perpendicular to the plane, so that the force balance is expressed by

$$K_0 = G - \Sigma F_a, \tag{1}$$

where K_0 is the interaction (reaction) force on the area of contact, and ΣF_a denotes the sum of all other forces, including dynamic forces.

The force balance (1) is known as the principle of d'Alambert. It is based on the laws of Newton, including his "*lex tertia*". According to equation (1), the body will adhere to the wall as long as the force K_0 is positive.

We suppose now that the force K_0 is positive ($K_0 > 0$) and assume we are able to change at least one of the forces F_a so that the right-hand-side in equation (1) becomes zero, $G - \Sigma F_a = 0$. Since at the same time the interaction force K_0 disappears, we have

$$K_0 = 0 \tag{2}$$

as the necessary conditions for a rigid body to leave the wall.

We can sum up our present results as: at the moment of detachment of a rigid body, the interaction (reaction) force K_0 between the body and the wall appearing in the force balance has to vanish. Or, in other words, the disappearance of the reaction force within the force balance, which is valid at all times, indicates the moment of detachment of the body. At this moment there exists a particular relationship between the forces acting on the body.

2.2. *Detachment of a vapour bubble of frozen shape*

We now focus our attention on a vapour bubble with concave shape that does not change during the detachment process, and state that a force balance based on the assumption of a frozen bubble shape during the bubble detachment leads to physical absurdity. To demonstrate this, we consider a vapour bubble of radius r surrounded by liquid, Fig. 2. For the reason of simplicity, but without any restriction of the results that follow, the action of gravity is excluded. The pressure within each phase is constant, the pressure in the vapour is different to that in the liquid.

One of the methods in finding out the relationship between the pressures p_G and p_L is a force balance. As shown in Fig. 2(a), the vapour in the upper half of the bubble is in equilibrium and the force balance leads to the well-known Laplace equation

$$p_G = p_L + 2\frac{\sigma}{r}. \tag{3}$$

Let us next attach the upper half of the bubble to a wall without distortion of equilibrium, that is, let us replace the cross-sectional area s–s in Fig. 2(a) by a real surface of a rigid wall, Fig. 2(b). The vapour in the bubble exerts, in this case, pressure on the wall and according to equation (3), the pressure in the vapour is thus higher than in the liquid.

We now consider the detachment of the bubble shown in Fig. 2(b) with the requirement of no change of bubble shape during the detachment process. In order to detach the bubble, a certain force F has to be assumed to act on the vapour, as shown in Fig. 2(c), and our task is to determine this force on the basis of a force balance.

The force balance for the system in Fig. 2(c) is

$$F + K_0 = p_L r^2 \pi + 2r\pi\sigma, \tag{4}$$

where K_0 denotes the interaction (reaction) force on the contact area between vapour and wall.

Since the shape of the bubble interface in Fig. 2(c) is the same as in Fig. 2(b), the pressure p_G in the vapour remains unchanged. Thus

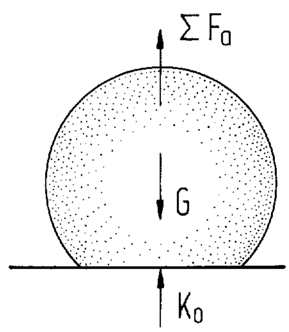

Fig. 1. Detachment condition of a rigid body.

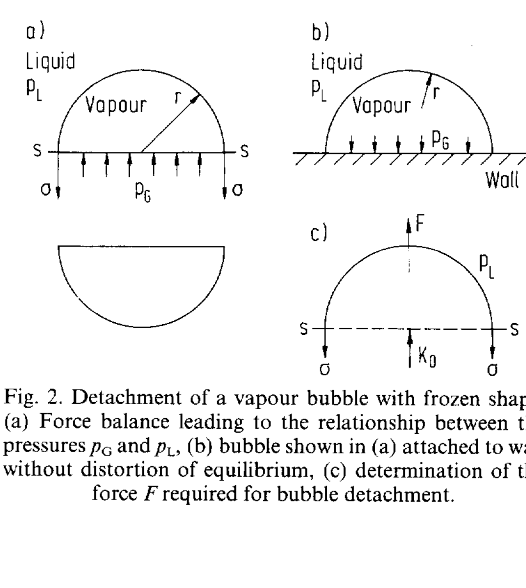

Fig. 2. Detachment of a vapour bubble with frozen shape. (a) Force balance leading to the relationship between the pressures p_G and p_L, (b) bubble shown in (a) attached to wall without distortion of equilibrium, (c) determination of the force F required for bubble detachment.

$$K_0 = p_G r^2 \pi. \tag{5}$$

A combination of equations (4) and (5) gives

$$F = \left(p_L - p_G + 2\frac{\sigma}{r}\right) r^2 \pi. \tag{6}$$

It is at this point where further reference to explanations in van der Geld's paper [2] should be made. According to van der Geld, the pressure difference $p_G - p_L$ which causes a buoyancy effect has to exist also at the moment of bubble detachment. This requirement leads to an unacceptable result, as immediately follows.

Remembering that the pressure p_G in Fig. 2(a, b) is the same, the term in the parentheses in equation (6) is zero, as required by equation (3). Thus

$$F = 0! \tag{7}$$

According to this result, no force is needed for the detachment of a vapour bubble under conditions of zero-gravity, and the bubble is expected to leave the wall spontaneously. This naturally contradicts our experience. As is well known from numerous boiling experiments under micro-gravity conditions, vapour bubbles do not detach, they, on the contrary, remain on the heating surface.

Equation (7) is, of course, erroneous not because of the exclusion of gravity action, but because the detachment condition has not been specified and included in the force balance. This error should be corrected as follows:

As explained in ref. [1], we first recognize that the difference

$$K_0 - p_L r^2 \pi$$

is the force acting on the vapour–wall contact area relative to the surrounding liquid, which is thus an excess force. As in ref. [1], we designate this force by ΔK. Thus, regarding equation (5), we arrive at

$$\Delta K = K_0 - p_L r^2 \pi = (p_G - p_L) r^2 \pi. \tag{8}$$

Using this relation, equation (4) can now be rearranged to give

$$\Delta K = 2r\pi\sigma - F. \tag{9}$$

According to our assumption, the bubble considered should not change its shape during detachment. The bubble detaches itself like a rigid body so that the excess force ΔK has to disappear at the moment of detachment, $\Delta K = 0$. From equation (9), we then obtain

$$F = 2r\pi\sigma. \tag{10}$$

This equation gives the correct results as far as the frozen bubble shape is concerned. As expected, the force needed for bubble detachment is not zero. However, this relation can hardly be used in connection with the real detachment process. As explained in ref. [1], the bubble does not detach from a wall leaving its shape unchanged, even the shape change manifests the bubble detachment.

To emphasize the physical absurdity arising from the assumption of the frozen bubble shape during the detachment, we combine equations (3) and (8) and write the excess force ΔK as

$$\Delta K = 2\frac{\sigma}{r} r^2 \pi. \tag{11}$$

In order to satisfy the detachment criterion ($\Delta K = 0$), the Laplace pressure $2\sigma/r$, or the contact area $r^2\pi$ has to disappear. However, the contact area cannot vanish due to our assumption. Since the radius r determining this area is responsible for the Laplace pressure, the Laplace pressure does not vanish either. Obviously, the detachment criterion is not satisfied and the bubble cannot detach itself.

The above considerations reveal that the Laplace pressure can indeed never be zero for a bubble of concave shape, as van der Geld stated correctly. His statement, however, needs to be supplemented in so far as the bubble of such a shape cannot detach from the wall.

3. CONCLUSIONS

(1) As explained in ref. [1], the shape of a vapour bubble undergoes a strong change during the detachment process. A vapour bubble of frozen shape cannot leave the wall. Actually, the bubble detachment manifests itself by a necking process. The detachment process comes to an end when the concave radius of the bubble neck becomes zero.

(2) If one assumes—in contrast to physical reality—the bubble shape to be frozen during the detachment process, the detachment criterion formulated in ref. [1] is correct. It states that the interaction force on the contact area between the vapour and wall has to vanish at the moment of detachment. This is also demonstrated in the present paper.

(3) The force balance given in ref. [1] is correct. The so-called buoyancy term has been taken into account. To show this, one can very easily express the excess force ΔK in equation (27) in ref. [1] using the Laplace pressure and get at once equation (11) in ref. [2]. Van der Geld's criticism is therefore unfounded. However, if we assume the bubble shape to be frozen, the excess force ΔK has to vanish at the moment of detachment.

(4) The van der Geld paper does not improve or refine our knowledge about bubble detachment. On the contrary, it is, in my opinion, rather a step back in the effort to throw some more light on the nature of bubble detachment. The only contribution of his paper seems to lie in keeping the questions associated with detachment of capillary bodies open for further discussions.

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